in the AIAA Reno Conference paper.[3] Even when the equation was corrected, it appeared that the series expansion had poor convergence, so that the linearized result was limited to small blockage parameters of order $\varepsilon C_{\mathrm{Du,sep}}(S/C) \leq 0.1$; ε is Maskell's blockage constant and has a value near 2.5 for three-dimensional flows. I would suggest the following changes.

Equation (14) in Ref. 2 should be corrected to read

$$\Delta C_{\mathrm{DM1}} = C_{\mathrm{DcM1}} - \frac{C_{\mathrm{Du,sep}}}{[1 + \varepsilon C_{\mathrm{DcM1}}(S/C)]} \quad \text{for} \quad \varepsilon C_{\mathrm{DcM1}}(S/C) \leq 0.1 \tag{1}$$

where

$$C_{\mathrm{DcM1}} = \frac{C_{\mathrm{Du,sep}}}{1 + \varepsilon C_{\mathrm{Du,sep}}(S/C)} \tag{2}$$

However, the full expression of Ref. 1 is preferred because of its broad applicability. In Ref. 1, the incremental drag coefficient due to wake constraint was found to be the solution of

$$\Delta C_{\mathrm{DM1}}^2 - \left[\frac{1}{\varepsilon(S/C)} + 2C_{\mathrm{DcM1}}\right]\Delta C_{\mathrm{DM1}} - C_{\mathrm{DcM1}}(C_{\mathrm{Du,sep}} - C_{\mathrm{DcM1}}) = 0 \tag{3}$$

where C_{DcM1} is obtained from Eq. (2).

The solution can be written as the negative root (because $\Delta C_{\mathrm{DM1}} < 0$),

$$\Delta C_{\mathrm{DM1}} = \frac{1}{2}\left(\left[\frac{1}{\varepsilon(S/C)} + 2C_{\mathrm{DcM1}}\right] - \left\{\left[\frac{1}{\varepsilon(S/C)} + 2C_{\mathrm{DcM1}}\right]^2 + 4C_{\mathrm{DcM1}}(C_{\mathrm{Du,sep}} - C_{\mathrm{DcM1}})\right\}^{\frac{1}{2}}\right) \tag{4}$$

The two-step drag coefficient correction finally becomes

$$C_{\mathrm{DcM2}} = \frac{(C_{\mathrm{Du,sep}} + \Delta C_{\mathrm{DM1}})}{(q_c/q)_2} = \frac{(C_{\mathrm{Du,sep}} + \Delta C_{\mathrm{DM1}})}{[1 + \varepsilon(C_{\mathrm{DcM1}} - \Delta C_{\mathrm{DM1}})(S/C)]} \tag{5}$$

A comparison of the full and the corrected linearized drag increments for a family of normal flat plates of area ratios up to $(S/C) = 0.24$ is presented in Fig. 1.

Figure 2 shows the improved effectiveness of the full two-step procedure through a comparison of the drag coefficients measured on normal flat plates of area ratio up to $S/C = 0.24$. The two-step method produces a nearly constant drag coefficient with increasing blockage, whereas Maskell's single-step approach overcorrects at large blockage.

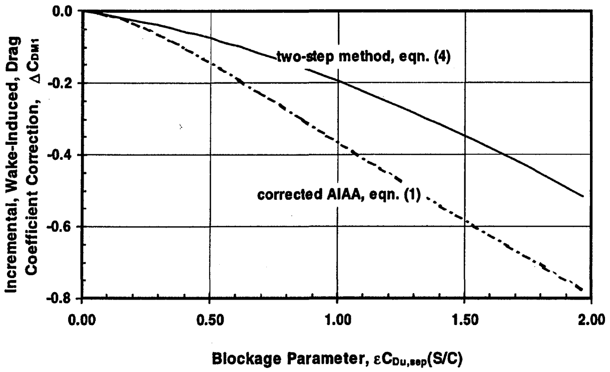

Fig. 1 Comparison of full and linearized incremental drag equations based on measurements on normal flat plates.

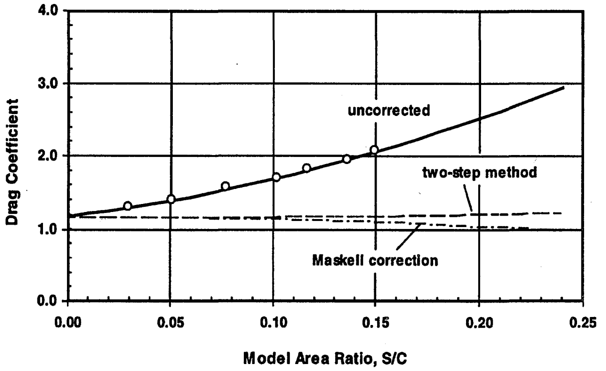

Fig. 2 Comparison of single-step and two-step correction procedures.

References

[1]Hackett, J. E., "Tunnel-Induced Gradients and Their Effect on Drag," Lockheed Martin Corp., Lockheed Engineering Rept. LG83ER0108, Revision 1, Marietta, GA, 1994.

[2]Hackett, J. E., "Tunnel-Induced Gradients and Their Effect on Drag," *AIAA Journal*, Vol. 34, No. 12, 1996, pp. 2575–2581.

[3]Hackett, J. E., "Tunnel-Induced Gradients and Their Effect on Drag," AIAA Paper 96-0562, Jan. 1996.

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Reply by the Author to K. R. Cooper

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Mr. Cooper is thanked for pointing out the typographical error in Eq. (14) of the *AIAA Journal* paper and the fact that the applicability of this equation is limited. His equations expose two significant changes that occurred during preparation of the conference version of the paper. The first is the use of the full quadratic solution for ΔC_{DM1}. The second is the introduction of ΔC_{DM1} in the denominator of his Eq. (5). Both were included in the original work of Ref. 1, from which the conference and the *AIAA Journal* figures were taken. Cooper makes a good point that the revised approach removes the tendency of the original Maskell form to overcorrect all forces and moments, not just drag.

In preparing the present response, it was found that, on substituting Cooper's Eq. (2) into Eq. (4), a simpler expression is obtained, namely,

$$\Delta C_{\mathrm{DM1}} = \frac{C_{\mathrm{Du}}}{1+x} + \frac{C_{\mathrm{Du}}}{2x}(1 - \sqrt{1+4x})$$

where $x = \epsilon C_{\mathrm{Du}} S/C$. The first term is Maskell's original correction, C_{DcM1} in the present discussion. The second term, which is negative and larger, is also a drag coefficient. It is indeterminate as x approaches zero but may be shown to equal zero in the limit. Expanding the preceding expression about $x = 0$ showed cancellation of the first three terms, in x^{-1}, x^0, and x, and a halved fourth term. It was found, also, that x reaches unity at $S/C = 0.165$ in Cooper's example. These features explain the poor convergence noted by him. Linearization is clearly hazardous in studies of the present type.

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Having found ΔC_{DMI}, Cooper's Eq. (5) completes the correction. An alternative, analytically equivalent form appears as Eq. (12) in Ref. 1, namely,

$$C_{\mathrm{DcM2}} = \frac{(C_{\mathrm{DcMI}} - \Delta C_{\mathrm{DMI}})(C_{\mathrm{Du}} + \Delta C_{\mathrm{DMI}})}{C_{\mathrm{Du}}}$$

It was also found that the mean of Eqs. (13) and (15), as printed in the *AIAA Journal* paper, provides a very close approximation to the correct result, provided that ΔC_{DMI} is calculated as shown here.

Reference

[1]Hackett, J. E.,"Tunnel-Induced Gradients and Their Effect upon Drag," Lockheed Martin Corp., Lockheed Engineering Rept. LG83ER0108, June 1983 (revision of Sept. 1994 available from the author).

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Comment on "Tunnel-Induced Gradients and Their Effect on Drag"

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In the December 1996 issue of the *AIAA Journal*, Hackett[1] discusses a wake-induced drag increment that is independent of the buoyancy drag, gradient times volume, form regularly used to correct wind tunnel data.[2] The obtained result is essentially correct, but because it is derived from kinematic concepts using local velocity gradients, it may lead to an erroneous interpretation that the buoyancy drag has already been accounted for in the procedure.[1]

Hackett's result can be identified more rigorously if derived instead from the balance of streamwise momentum. Let us use U for axial velocity, ρ for density, and p for pressure and denote further by C and Q the cross-sectional areas of the test section and wake, respectively (Fig. 1). Conservation of mass and energy between an upstream station 1 and a downstream station 2 gives

$$CU_1 = (C - Q)U_2 \tag{1}$$

$$\tfrac{1}{2}\rho U_1^2 + p_1 = \tfrac{1}{2}\rho U_2^2 + p_2 \tag{2}$$

The accompanying momentum relationship, based on the assumption that there is constant pressure across station 2 while flow takes place only outside the wake, is

$$C\left(\rho U_1^2 + p_1\right) = (C - Q)\rho U_2^2 + Cp_2 + F \tag{3}$$

The isolated force term $F > 0$ compensates for the momentum deficit at station 2. Its presence reflects the fact that, between stations 1 and 2, transfer of momentum from fluid to the model takes place, giving rise to an incremental drag. The correction to the model drag force is thus

$$\Delta D = -F = -\tfrac{1}{2}\rho U_1^2 C[Q/(C - Q)]^2 \tag{4}$$

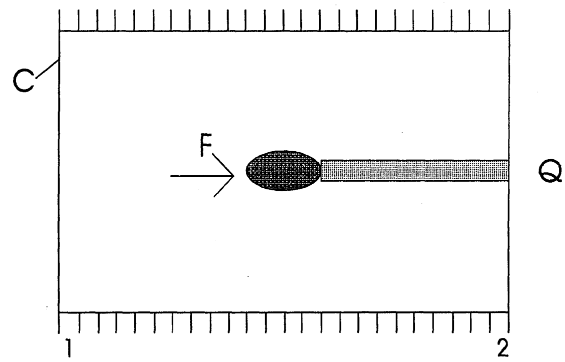

Fig. 1 Model representation in the wind tunnel.

as may be derived from Eqs. (1–3). Assuming $Q \ll C$, introducing the wake source strength

$$Q_W = D/(\rho U_1) = U_1 Q \tag{5}$$

and the force coefficient reference area S, we obtain from Eq. (4) the drag coefficient correction

$$\Delta C_D = \frac{2\Delta D}{\rho U_1^2 S} \simeq -\frac{C}{S}\left(\frac{Q_W}{U_1 C}\right)^2 \tag{6}$$

in agreement with Hackett[1] [Eq. (11a), p. 2581].

Because $\Delta C_D \to 0$ as $C \to \infty$ or $Q_W \to 0$ and the way it has been derived, Eq. (6) represents the correction to the drag coefficient for the wall-induced change of stream momentum. There is no direct connection to the model volume and no accounting for the buoyancy effect. In view of the fact that the formula has been derived using the conservation equations (1–3), the latter observation may appear paradoxical but of course only in the sense of d'Alembert. The inviscid drag force on an isolated system (model and walls) must namely be zero, with no net effect on the streamwise momentum of the surrounding fluid. Admittedly, if the wind tunnel walls are parallel and impermeable (no transfer of momentum), there can be no axial force on the walls and no axial force on the model either. However, once the wake enters the picture, a streamwise pressure gradient that is similar to that generated by convergent wind tunnel walls will be generated. Because the origin of the pressure gradient makes little difference, the buoyancy force on the model may be regarded as equal and opposite to that on the convergent, fictitious walls. In reality, the walls are parallel, but there is a third player facilitating the counterbalance: the displacement body of the wake.

Hackett's main contribution is in deriving and experimentally verifying a wall interference correction to the measured drag coefficient and comparing it with an earlier, semiempirical approach by Maskell.[3] The introduced correction formula applies universally to all types of models (bluff or streamlined) if tested at low speeds in solid-wall wind tunnels. It accounts for the wall-induced momentum deficit, corresponding to the upstream–downstream pressure difference, not buoyancy, corresponding to the pressure gradient at the model.

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